Unit 12 Optimization

12-1 - Applied Max/Min Problems

__Objective__ - You will use the derivative to solve applied (Real life) Max/Min Problems.

__Applied Max/Min Problems__

1) Identify the variables with let statements or draw a diagram.

2) Set up an equation based on the given info and solve for y.

3) Set up an equation to be maximized/minimized.

4) Take the first derivative of the equation from step 3, set = 0 and solve for x.

5) Check that your answer is a max/min using a number line analysis with the first derivative.

__Ex__ 1) Find two positive numbers whose sum is 20 and whose product is a maximum.

Let x = 1st number
Let y = 2nd number

$x + y = 20$ $P = x \cdot y$

$y = -x + 20 \longrightarrow P = x(-x+20)$

$P = -x^2 + 20x$

$P' = -2x + 20$

$-2x + 20 = 0$

$\frac{-2x}{-2} = \frac{-20}{-2}$

$\boxed{x = 10}$

$\longleftarrow + + + + + + \mid - - - - - - - \longrightarrow P'$ (10)

So @ x=10 MAX

- or -

$P'' = -2$ (Concave Dn)

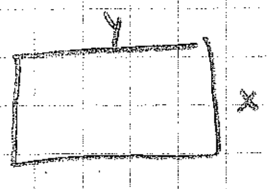
∴ Check endpoints

$x = 1$ $x = 19$

$y = 19$ $y = 1$

So the #'s are 1 and 19.

2) A rectangle has a perimeter of 100 feet. What length and width should it have so that its area is a maximum.



let x = width of □
let y = length of □

$2x + 2y = 100$

$2y = -2x + 100$

$y = -x + 50$

$A = x \cdot y$

$A = x(-x + 50)$

$A = -x^2 + 50x$

$A' = -2x + 50$

$-2x + 50 = 0$

$\frac{-2x}{-2} = \frac{-50}{-2}$

$\boxed{x = 25}$

$y = -25 + 50$

$\boxed{y = 25}$

$\xleftarrow{+ + + + + +} \; | \; \xrightarrow{- - - - - -} \; A'$

25

– or –

$A'' = -2$ (Concave Dn)

Max @ $x = 25$

$\boxed{\therefore x = 25 \quad y = 25}$

HW: WS #1, 2

Applied Max / Min Problems

1. Find two positive real numbers whose sum is 40 and whose product is a maximum.

2. Find two positive numbers that minimize the sum of twice the first number plus the second if the product of the two numbers is 288.

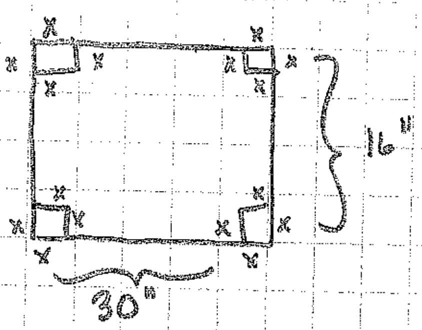
3. The sum of two numbers is 16. Find the numbers if the sum of their cubes is a minimum.

4. A farmer wishes to enclose three identical adjacent rectangular areas, each with 900 sq. ft. of area. What dimensions should be used to minimize the amount of fence required.

5. An open box having a square base is to be constructed from 108 sq. in. of material. What should be the dimensions of the box to obtain a maximum volume?

6. A poster is to contain a printed area of 150 sq. in., with clear margins of 3 in. top and bottom, and 2 in. on each side. Find the minimum total area.

3) An open box is to be made from a 16" x 30" piece of Cardboard by cutting out squares of equal size from the 4 corners and bending up the sides. What size should the squares be to obtain a box with the largest possible volume?



Let x = side of □

$V = l \cdot w h$

$V = (30-2x)(16-2x)(x)$

$V = (480 - 60x - 32x + 4x^2)\,x$

$V = (480 - 92x + 4x^2)\,x$

$V = 4x^3 - 92x^2 + 480x$

$V' = 12x^2 - 184x + 480$

$V' = 4(3x^2 - 46x + 120)$

$V' = 4(3x-10)(x-12)$

$0 = 4(3x-10)(x-12)$

$4 \neq 0$ | $3x - 10 = 0$ | $x - 12 = 0$

$\frac{3x}{3} = \frac{10}{3}$ $\boxed{x = 12}$ Too Big!

$\boxed{x = 10/3}$

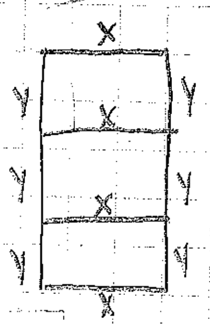
+++++++ ------ V'

10/3

$\boxed{\text{Max @ } x = 10/3}$

ON WS

4)



$x \cdot y = 900$

$y = \frac{900}{x}$

Let x = the length $\boxed{= 15\sqrt{6}}$

Let y = the width $\boxed{= 10\sqrt{6}}$

+++++ ------

$15\sqrt{6}$

min @ $x = 15\sqrt{6}$

$F = 4x + 6y$

$F = 4x + 6\left(\frac{900}{x}\right)$

$F = \frac{4x}{1} + \frac{5400}{x}$

$F = \frac{4x^2 + 5400}{x}$ $f = 4x^2 + 5400$ $g = x$

$f' = 8x$ $g' = 1$

$F' = \frac{x(8x) - (4x^2 + 5400)(1)}{x^2}$

$F' = \frac{8x^2 - 4x^2 - 5400}{x^2}$

$F' = \frac{4x^2 - 5400}{x^2}$

$F' = \frac{4(x^2 - 1350)}{x^2}$

$\frac{0}{1} = \frac{4(x^2 - 1350)}{x^2}$

$4(x^2 - 1350) = 0$

$x^2 - 1350 = 0$

$x^2 = 1350$

$x = \sqrt{225}\sqrt{6}$

$\boxed{x = 15\sqrt{6}}$

$y = \frac{900}{15\sqrt{6}} = \frac{60}{\sqrt{6}} = 10\sqrt{6}$ $\boxed{10\sqrt{6}}$

Homework: The sum of two positive numbers is 40. Find these numbers if the product of the cube of the first number times the second is a maximum.

#3 on Worksheet The sum of two numbers is 16. Find the numbers if the sum of their cubes is a minimum

Let x = 1st #
Let y = 2nd #

$x+y=16$
$y=-x+16$

$S = x^3+y^3$
$S = x^3+(-x+16)^3$

$S' = 3x^2+3(-x+16)^2 \cdot (-1)$

$S' = 3x^2+-3(-x+16)^2$

$S' = 3x^2-3(-x+16)(-x+16)$
$S' = 3x^2-3(x^2-32x+256)$
$S' = 3x^2-3x^2+96x-768$

$S' = 96x-768$ *

$96x-768=0$
$\frac{96x}{96}=\frac{768}{96}$
$x=8$

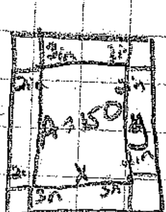
$\longleftarrow - - - - \underset{8}{|} + + + + + \longrightarrow S'$

Min @ x=8

$y=-8+16=8$

$\boxed{\therefore \text{ 8 and 8 are the \#'s}}$

#6 on WS.



Let $x+4$ = side
$y+6$ = top/bot

$x \cdot y=150$
$y=\frac{150}{x}$ or $150x^{-1}$

$A=(x+4)(y+6)$
$A=(x+4)(150x^{-1}+6)$
$A=150+6x+600x^{-1}+24$
$A'=6+-600x^{-2}$

$A'=6-\frac{600}{x^2}$

$0=6-\frac{600}{x^2}$

$\frac{6}{1}=\frac{600}{x^2}$

$\frac{6x^2}{6}=\frac{600}{6}$

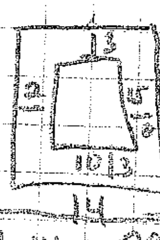
$x^2=100$ $\boxed{x=10}$

$\longleftarrow - - - \underset{10}{|} + + + \longrightarrow A'$

Min @ x=10

$y=\frac{150}{10}$

$y=15$



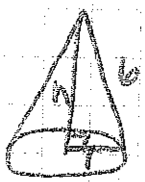
21

14

$\boxed{21 \times 14 = 294}$

HW #5

You are to create an ice cream cone with a slant height of 6 inches. What should the height and radius of the cone be if you want to maximize the volume?



$h^2 + r^2 = 6^2$

$h^2 + r^2 = 36$

$r^2 = 36 - h^2$

$\boxed{r = (36-h^2)^{1/2}}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left((36-h^2)^{1/2}\right)^2 \cdot h$

$V = \frac{1}{3}\pi(36-h^2)\cdot h$

$V = \frac{1}{3}\pi(36h - h^3)$

$V = 12\pi h - \frac{\pi h^3}{3}$

$V' = 12\pi - \pi h^2$

$12\pi - \pi h^2 = 0$

$-\pi h^2 = -12\pi$

$\frac{-\pi h^2}{-\pi} = \frac{-12\pi}{-\pi}$

$\sqrt{h^2} = \sqrt{12}$

$h = \sqrt{4}\sqrt{3}$

$\boxed{h = 2\sqrt{3}}$

$+ + + + \mid - - - - -$

$2\sqrt{3}$

Max @ $\boxed{h = 2\sqrt{3}}$

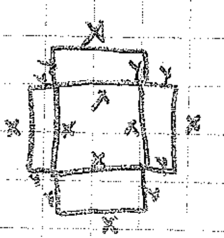
$r = (36 - (2\sqrt{3})^2)^{1/2}$

$r = \sqrt{24}$

$r = \sqrt{4}\sqrt{6}$ $\boxed{r = 2\sqrt{6}}$

∴ The height should be $2\sqrt{3}$ in and radius is $2\sqrt{6}$ in

The total surface area of a squared base open top rectangular box is 12 units². Find the dimensions of the box such that the volume is a maximum.



A = square + 4 rect.

$A = x^2 + 4xy$

$12 = x^2 + 4xy$

$12 - x^2 = 4xy$

$y = \frac{12 - x^2}{4x}$

$\boxed{y = 3x^{-1} - \frac{1}{4}x}$

$V = x^2 \cdot y$

$V = x^2\left(3x^{-1} - \frac{1}{4}x\right)$

$V = 3x - \frac{1}{4}x^3$

$V' = 3 - \frac{3}{4}x^2 = 0$

$-\frac{3}{4}x^2 = -3$

$x^2 = 4$ $x = \pm 2$ $\boxed{x = 2}$

$+ + + \mid - - -$

2

$y = 3(2)^{-1} - \frac{1}{4}(2)$

$\boxed{y = 1}$

Max @ $\boxed{x = 2}$

∴ The base is 2 and height is 1.